**20200929 Homework**

1. **Distance of discrete random variables.**

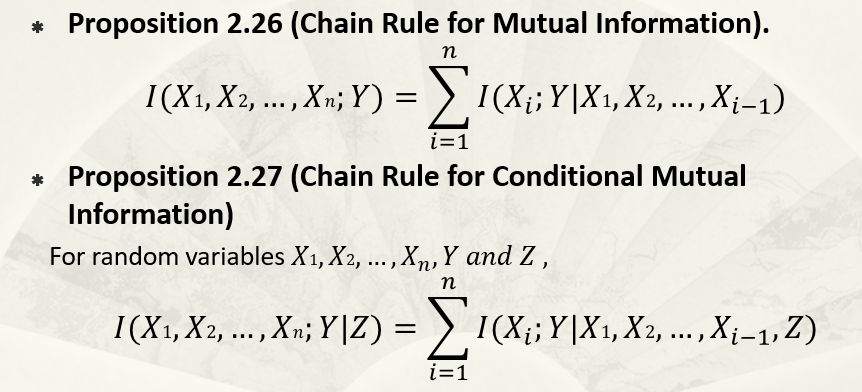
Prove that is a distances of discrete random variables *X* and *Y*.

令

1. since H(X|Y) >= 0 and H(Y|X)>=0 always hold.
2. .
3. H(X|Y) >= 0 and H(Y|X)>=0. If we say X=Y if there is a one-to-one mapping from X to Y, H(Y|X)=H(X|Y)=0 holds.
4. ,,,

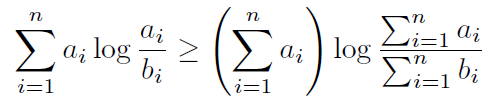
By definition, we have , and ,. So .

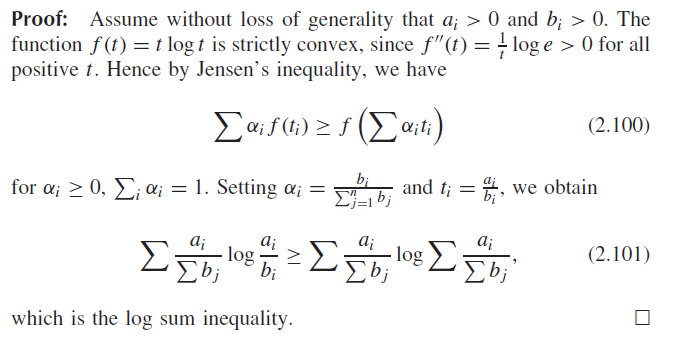
**2. Prove the following two chain rules.**



**条件互信息同理**

**3. Log-Sum inequality**: For non-negative numbers and , prove

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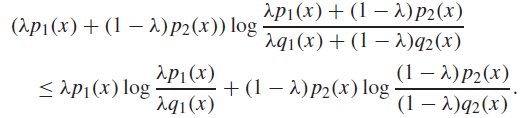
**4. Convex Relative Entropy**

If and are pairs of probability mass functions then



for all . That is, is **convex function** of the pair .

For any fixed x, applying log sum inequality yields



Summing over all x, the proof is completed.

**5. Concave Entropy**

Let be the probability mass function of discrete random variable *X*. Here is denoted by . Prove that

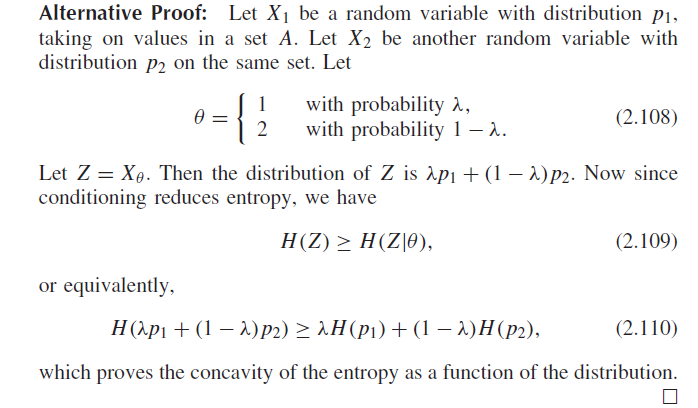


That is, is a **concave function** of .

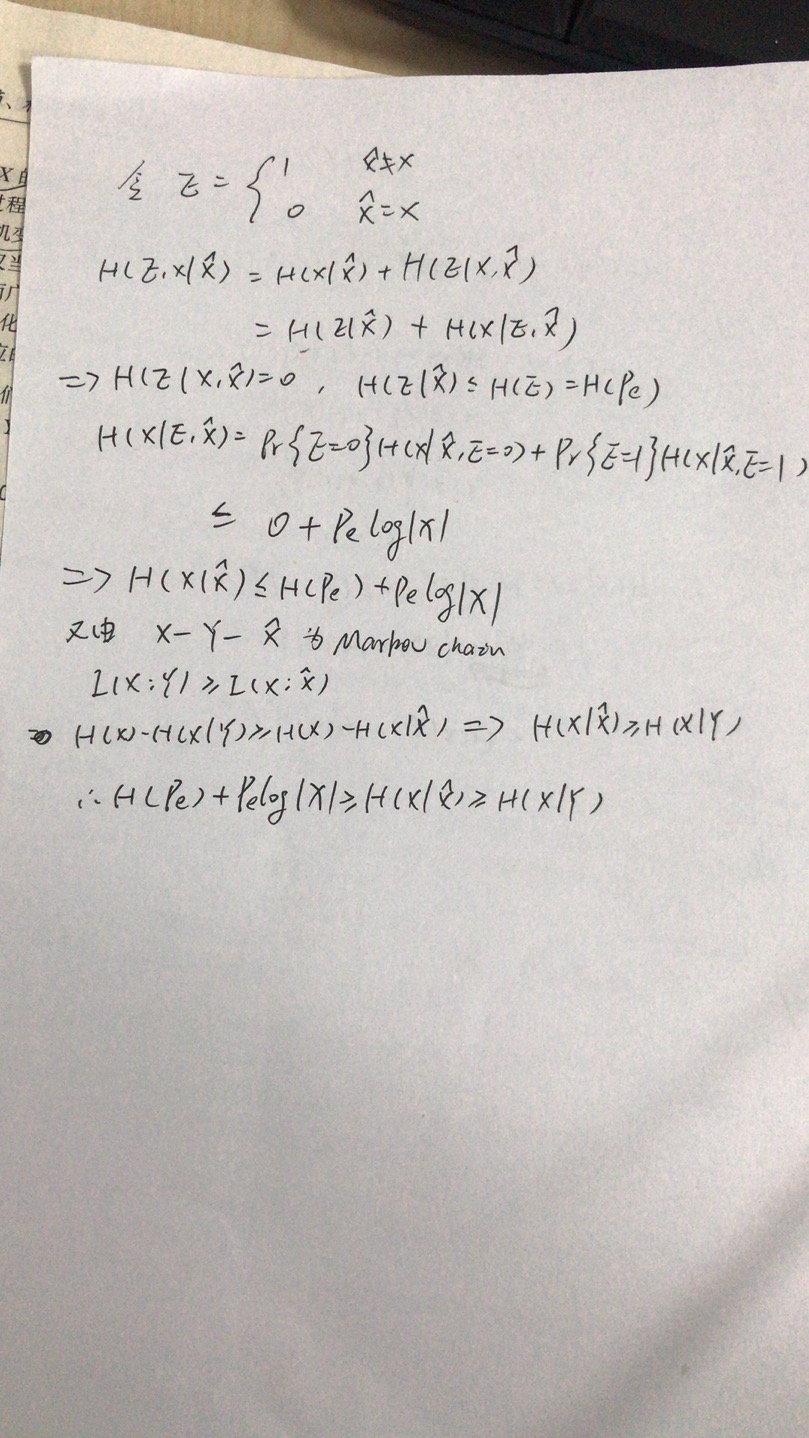
Let be the uniform distribution on alphabet .

So The concavity of H then follows directly from the convexity of D.

另一种做法



**6. Prove the Fano’s inequality.**

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**7.** If is a Markov chain, **judge and show** whether is also a Markov chain.

1) When =0， is also a Markov chain.

2）Otherwise, it is not a Markov chain.

**8. Remark. Convex and Concave mutual information**

Mutual Information can be expressed by a function of input distribution and transition distribution , i.e.,

.

1. For given input distribution , we say that is **convex** of transition distribution .
2. For given transition distribution, we say that is **concave** of input distribution .

**Convex 8A can be explained by:**

Let and be two joint distributions, we define

**Then, for given input distribution , is convex of transition distribution , i.e.,**

**i.e.**